

# A Generalized Random Mobility Model for Wireless Ad Hoc Networks and its Analysis: One-Dimensional Case

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**Abstract**—In wireless ad hoc networks, the ability to analytically characterize the spatial distribution of terminals plays a key role in understanding fundamental network QoS measures such as throughput per source to destination pair, probability of successful transmission, connectivity, etc. Consequently, mobility models that are general enough to capture the major characteristics of a realistic movement profile, and yet are simple enough to mathematically formulate its long-run behavior, are highly desirable.

In this paper, we propose a generalized random mobility model capable of capturing several mobility scenarios and give a mathematical framework for its exact analysis over one-dimensional mobility terrains. The model provides the flexibility to capture hotspots where mobiles accumulate with higher probability and spend more time. The selection process of hotspots is random and correlations between the consecutive hotspot decisions are successfully modeled. Furthermore, the times spent at the destinations can be dependent on the location of destination point, the speed of movement can be a function of distance that is being traveled, and the acceleration characteristics of vehicles can be incorporated into the model. Our solution framework formulates the model as a semi-Markov process using a special discretization technique. We provide long-run location and speed distributions by closed-form expressions for one-dimensional regions (e.g., a highway).

**Index Terms**—Mobility Modeling, Long-Run Analysis, semi-Markov Processes, Ad Hoc Networks

## I. INTRODUCTION

WIRELESS ad hoc networks are comprised of wireless mobile nodes that can dynamically form a network in a self-organizing manner without the need for a pre-existing fixed infrastructure. Nodes in an ad hoc network can move according to many different mobility profiles. Therefore, mobility models that dictate the movement behavior of a mobile terminal play a key role in the simulation or analytical based analysis of the impact of dynamically changing topology on the performance of these networks. In this paper, we consider a generalized random mobility model that is flexible enough to capture different mobility scenarios, and provide its long-run location and speed distributions by closed form expressions for one-dimensional mobility terrains.

In what follows, we categorize the existing mobility models for wireless ad hoc networks, and briefly summarize their assumptions. Traditionally, a mobility model governs the

changes in the moving direction and speed of terminals according to a deterministic approach or a random process. In the former case, movement path of terminals can be restricted to predetermined paths. For ad hoc environments, such mobility models are impractical since wireless ad hoc networks are created “on the fly”, and collecting data to generate the paths for all situations can be very complicated. Thus, a mobility model that dictates the movement of hosts due to a random process, that is, *random mobility model*, is more appropriate for the performance evaluation of these networks. Surveys for both models are presented in [1], [2].

In general, random mobility models formulate the movement pattern of mobile hosts by consecutive random length intervals called movement epochs. During each epoch, mobile terminal moves at a constant speed, and at a constant direction for a random amount of time. The speed and direction choice for each epoch may or may not be correlated with their values in the previous epochs, and mobility characteristics of other terminals. For instance, according to the random walk mobility model [2], each terminal movement is uncorrelated with other’s movement, and the speed and direction choices for each epoch are also uncorrelated with their previous choices. The random waypoint mobility model [3] includes pauses at the end of movement epochs in the random walk model to make it more applicable to different scenarios. More formally, according to the random waypoint mobility model, a mobile node determines a destination point that is distributed *uniformly* within the physical terrain and moves in the direction of that destination at a constant speed. This speed is selected uniformly from  $[v_{\min}, v_{\max}]$  where  $v_{\min} > 0$ , and it is *independent* from the destination and starting points of the movement epoch, and also the distance that is going to be traveled. After reaching the destination, mobile pauses for a random amount of time, which has the *same distribution* for all destination points, and the same movement process is repeated by selecting a new destination and speed pair *independently* from the same pair of the previous movement epoch.

A shortcoming of the random mobility models is that the movement profiles that are generated with respect to them may not be consistent with the major characteristics of a realistic scenario. For instance, as it also mentioned in [1], the random walk and the random waypoint mobility models may generate unrealistic movement patterns such as “sudden stops” and “sharp turns”. In [4], [5], [6], authors propose models that can capture correlation between the speed and the direction choices of consecutive movement epochs and therefore these models may generate a pattern which is smoother with less sharp turns. Furthermore, as it is also criticized in [7], [8], selecting speed independently from the distance that is going

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to be traveled may end up in unrealistic mobility profiles where mobiles travel long distances with low speeds.

A common limitation of the random mobility models described above is that one cannot model a scenario which incorporates predefined pathways that mobiles must follow and specific destinations on those paths where mobiles accumulate with higher probability. The models presented in [9], [10] focus on this problem by taking a more deterministic approach that can capture obstacles and predefined pathways between them on the physical terrain.

In the analytical studies for the performance analysis of wireless ad hoc networks, closed form expressions for the spatial node distribution are very desirable to understand long-run behavior of the network spatial behavior. For instance, the analyses that are presented in [11], [12], [13] to estimate the capacity per source to destination pair of these networks are significantly dependent on the spatial distribution of mobile nodes. Additionally, for some scenarios in which terminals can be highly mobile on a wide region, the spatial distribution of offered traffic may not be ignored in determining the capacity of asynchronous MAC layer protocols. Observe that the analysis of this case requires an accurate knowledge of the spatial distribution of nodes. Also, the analytical work presented in [14] considers the station locations for the MAC layer throughput analysis but the terminals are assumed to be uniformly distributed in the region. Clearly, the uniform distribution assumption may not be valid for different mobility scenarios. Moreover, this knowledge can be also used in evaluating the connectivity properties of ad hoc networks, which have been extensively studied in [15], [16]. In addition to these, the distribution of link distance between mobile terminals, which is an important characteristic of wireless ad hoc networks [17], [18], can be obtained from the spatial distribution of terminals.

Hence in this paper we propose a generalized random mobility model that is general enough to capture the major characteristics of a realistic movement profile, and yet is simple enough to mathematically formulate its long-run behavior with analytical expressions. The mobility pattern of a terminal that moves according to this generalized model is composed of consecutive movement epochs in a closed region, and it is independent with the movement behavior of other terminals. During each movement epoch, mobile terminal firstly moves on the finite line segment joining the starting and destination points of the epoch at a random speed and then it pauses at the destination for a random amount of time. The generality of our model is actually originating from the approach that we took to determine the destination point, movement speed, and pause time at the destination, and can be explained as follows:

- The distribution of the destination points are assumed to be general and can be conditionally dependent on the starting point of the movement epoch.
- The random speed for each epoch is drawn from a general distribution function that can be conditionally dependent on the starting and destination locations of the movement epoch, and the current location of mobile terminal if necessary.
- The pause time at each destination is selected randomly

from a distribution that is dependent on the location of the destination point.

The fact that we make the mobility modeling with respect to these generalized approaches has a number of advantages. First, since destinations are selected from a general distribution, a movement scenario in which terminals select some specific locations, for example, *hotspots*, as destination with higher probability, can be easily captured. Furthermore, some mobility scenarios may require a Markovian dependency between the destination points of consecutive movement epochs. For instance, the probability of selecting a hotspot as destination can be different for different starting points. This case can be naturally incorporated into our model by employing a distribution function for destinations that is conditionally dependent on the starting points.

Second, the generic approach for determining speed provides a unique opportunity to select speed according to the distance that is going to be traveled, and also a method to model variable speed during movement epochs. Clearly, if the speed of the terminal can vary during moving, then our model can even be used to capture different *acceleration* characteristics of vehicles. Finally, by employing a pause time distribution for each epoch that is a function of destination coordinate, we reached to the flexibility of pausing different times at different locations.

For some sophisticated mobility models, performing its long-run analysis first over one-dimensional regions will be useful in gaining some insight into the methodology that has to be followed for the analysis of higher dimensions. Thus, in this paper, we concentrate our analysis to one-dimensional regions, and develop an analytical framework that provide closed form expressions for the long-run location and speed distributions. We also believe that the analytical results presented can provide a methodology to analytically formulate the fundamental properties of wireless ad hoc networks for number sophisticated mobility scenarios (e.g., capacity, connectivity).

#### A. Related work

There have been a number of works attempting to obtain spatial node distribution for the ad hoc environments where terminals move according to random walk or random waypoint mobility models. The simulation studies that are presented in [19] and [20] for the random waypoint mobility model showed that the long-run spatial distribution of mobiles is independent from their initial placement in the simulation area, and also observed that resulting distribution is more accumulated at the center of the region. In [21], the movement pattern of the same mobility model is characterized as a stochastic process, and analytical expressions for the long-run location distribution are derived. In [22], authors not only concentrate on the analytical expressions for long-run spatial distribution of random waypoint model, but also on the limiting distribution of speed and procedures for the accurate simulation of this mobility model as well. The simulation study presented in [7] also concentrated in the same model, and examined average node speed at the steady-state. They pointed out that the closer  $v_{\min}$  to zero, the more time it takes

for the simulation of the mobility model to reach stability. In [8], this work is extended by analytical studies and authors provided steady-state average speed distribution for several random mobility models in which the speed for a movement epoch is chosen independently from the destination of that epoch. For these mobility models, as a byproduct of their analytical formulation, authors also proposed a simulation methodology that decreases the variations in the average nodal speed. In [23], authors provide an analytical framework for the steady-state speed and residual distance analysis of random waypoint mobility model, and similar to [8], they also proposed methodology for the efficient simulation of this mobility model. In [24], a statistical analysis is done to identify the conditions in which the spatial node distribution of random waypoint mobility model, and a variant of two-dimensional random walk motion can be approximated with uniform distribution.

While each of the analytical and simulation studies mentioned above provide a comprehensive approach for the long-run characteristics of the random walk and random waypoint mobility models and their variants, none attempts to make major extensions on these models so that they describe a more realistic pattern. As it is also mentioned in [10], mobility models that includes predetermined pathways and obstacles are more realistic than the random mobility models. However, capturing these realistic issues requires to add detailed deterministic parameters to the mobility model and as the deterministic dimension of the mobility model expands, the possibility of deriving long-run properties of the model in terms of closed from expressions decreases. The most significant differences between the mobility model proposed in this paper and other random or deterministic models are the degrees of generality in mobility modeling and simplicity for the long-run analysis.

The next section provides the mobility formulation according to our mobility model, basic definitions, and our approach for long-run analysis. In the third section, the analytical results are presented with example scenarios. Section IV concludes the paper.

## II. MOBILITY FORMULATION

In this section, we provide the formal description of the generalized random mobility model introduced in Section I for one-dimensional mobility terrains, and construct an analytical framework for its long-run analysis. Let  $R = [0, a]$  represent the bounded region on which mobile terminals operate, and denote  $X_s \in R$  and  $X_d \in R$  as the random variables corresponding to the starting and destination points of a movement epoch, respectively. Furthermore, let the random variable  $V$  defined on the state space  $[v_{\min}, v_{\max}]$ , where  $v_{\min} > 0$ , denote the speed of a terminal while moving from  $X_s$  to  $X_d$ . In addition, denote the random variable  $T_p$  with state space  $[0, \infty)$  as the pause time spent at destination point  $X_d$ . With respect to these notations, and the mobility model we proposed in this paper, we define the following parameters:

- $f_{X_d|X_s}$ : the conditional probability density function (pdf) of  $X_d$  given  $X_s$ ,
- $f_{V|X_s, X_d}$ : the conditional pdf of  $V$  given  $X_s$  and  $X_d$ ,
- $f_{T_p|X_d}$ : the conditional pdf of  $T_p$  given  $X_d$ .

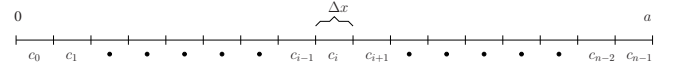


Fig. 1. Discretization of  $R = [0, a]$  according to cells of size  $\Delta x = \frac{a}{n}$ .

Hence, the mobility formulation that is performed according to the generalized random mobility model can be characterized by the triplet  $\langle f_{X_d|X_s}, f_{V|X_s, X_d}, f_{T_p|X_d} \rangle$ . Since we are concentrating on the long-run properties of the mobility model, the initial distribution of the terminals can be arbitrary.

Before we proceed further, we note that  $X_s$  and  $X_d$  actually represent the destination points of any two consecutive movement epochs, and the conditional pdf  $f_{X_d|X_s}$  that identifies the distribution of  $X_d$  given  $X_s$  at the embedded points in time where a new epoch starts, is referred as *stochastic density kernel* by Feller [25]. We will identify the restrictions on the choice of  $f_{X_d|X_s}$  required for the long-run characterization as we proceed further in the analysis.

Now as we have noted in Section I, each terminal's movement is assumed to be independent from others. Thus, it is enough to model a single terminal's behavior for the long-run analysis. For this purpose, let  $\mathbf{X}(t)$  denote the state of the mobile terminal at time  $t$ . According to the specifications of the mobility model we proposed, the stochastic process  $\{\mathbf{X}(t), t \geq 0\}$  must be defined on a state space that has separate dimensions for current location, destination, and speed, and more importantly, the ranges of these dimensions must be continuous. However, in the analytical framework we construct, we use a discretization method and describe the mobility behavior of nodes with a stochastic process that is defined on a multidimensional *discrete* state space. In addition, instead of observing the state of a terminal continuously, we will observe it at embedded times  $T_k$ , for  $k \in \mathbb{N}$ , such that  $T_0 = 0$ ,  $T_{k+1} \geq T_k$ ,  $\forall k \in \mathbb{Z}^+$ . Also, these embedded times are dependent on the evolution of the system that dictates the movement behavior of the mobile node. The following list formally defines the assumptions that the analytical framework is built on:

- $A_1$ : The region  $R$  is discretized into  $n$  cells of the same length, that are denoted by  $c_i = [(i-1)\Delta x, i\Delta x)$ ,  $i = 0 \dots n-1$ , as shown in Fig. 1, where  $\Delta x = \frac{a}{n}$  for  $n > 1$ . A mobile terminal is assumed to occupy one of the  $c_i$ 's at any moment in time, and movement epochs start from a cell and ends up at a different destination cell.
- $A_2$ : The random variable  $V$ , which denotes the speed of a mobile during a movement epoch, is approximated by the discrete random variable  $V^*$  taking values in the state space  $\mathcal{S}_{V^*} = \{z_1, z_2, \dots, z_m\}$ , where  $z_r = r\Delta v$ ,  $r = 1, \dots, m$ , for some discretization parameter  $\Delta v > 0$ , and  $m > 1$  such that  $\Delta v \leq v_{\min}$  and  $v_{\max} \leq m\Delta v$ .
- $A_3$ : Observation time  $T_k$  point to the time of occurrence of one of the following events:
  - $E_1$ : In pause mode, the terminal selects a new destination that is different from the current cell occupied, and jumps into moving state at the current cell,
  - $E_2$ : While traveling in the direction of the target cell, the terminal moves out from the current cell and enters the neighbor cell that lies on the shortest path joining the current and destination cells with the least number of cells.

$E_3$ : The terminal reaches to the destination cell and enters the pause mode at that location.

Notice that the higher the degree of discretization for the closed region  $R$  is selected, the better approximation can be done to the exact location of the terminals. Also, as the discretization parameter  $\Delta v \rightarrow 0$  (i.e.,  $m \rightarrow \infty$ ), the discrete approximating random variable  $V^*$  becomes indistinguishable from the original random variable  $V$ . Therefore, as  $\{n, m\} \rightarrow \infty$ , we converge to model with continuous state space. For the rest of this paper, we will use the term *discretized mobility formulation* to refer to the version of the generalized random mobility modeling approach that is constructed according to the assumptions  $A_1$ ,  $A_2$ , and  $A_3$ .

Now, let  $\mathbf{S}_k$ ,  $k \in \mathbf{N}$ , denote the state of the mobile terminal at time  $T_k$ . Given the assumptions  $A_1$ ,  $A_2$  and  $A_3$ , the finite-state space of  $\mathbf{S}_k$  will be defined as follows:

$$\begin{aligned} \mathcal{S} = & \{(c_i, c_j, z_r, q) \mid i, j = 0, \dots, n-1, i \neq j, \\ & r = 1, \dots, m, q = 1\} \\ \cup & \{(c_i, q) \mid i = 0, \dots, n-1, q = 0\} \end{aligned} \quad (1)$$

where  $c_i$  is the current cell occupied,  $c_j$  is the destination cell,  $z_r$  is the discretized speed, and  $q$  is the indicator of being in the mode of moving towards the target cell, or pausing at the destination.

Hence, the stochastic process  $\{\mathbf{X}(t), t \geq 0\}$  that represents the state of the mobile terminal at time  $t$ , can be redefined on the finite-state space  $\mathcal{S}$  by the following expression:

$$\mathbf{X}(t) = \mathbf{S}_k, \quad \text{if } T_k \leq t < T_{k+1}$$

where the times  $T_1, T_2, \dots$  are the successive times of transitions of  $\mathbf{X}(t)$ , and  $\mathbf{S}_0, \mathbf{S}_1, \mathbf{S}_2, \dots$  represent the successive states occupied by  $\mathbf{X}(t)$ .

Observe that by constructing a state space that has a separate dimension for the destination cell of moving terminals, the future evolution of the stochastic process  $\{\mathbf{S}_k, k \in \mathbf{N}\}$  becomes dependent only on the current state of the mobile terminal, not on its history at previous observation points. Furthermore, assume that current state occupied by  $\mathbf{X}(t)$  is  $s$ . Once the state  $s' \in \mathcal{S}$  has been selected with some probability as the next state to be visited, the distribution of sojourn time in state  $s$  can be determined from the components of state  $s$ . Consequently, the following relationship will be valid for all  $k \in \mathbf{N}$ , and all possible sets  $\{s, s'\} \subset \mathcal{S}$ .

$$\begin{aligned} \Pr\{\mathbf{S}_{k+1} = s', T_{k+1} - T_k \leq t \mid \mathbf{S}_k = s, T_k, \dots, \mathbf{S}_0, T_0\} \\ = \Pr\{\mathbf{S}_{k+1} = s', T_{k+1} - T_k \leq t \mid \mathbf{S}_k = s\} \end{aligned}$$

Therefore, the stochastic process  $\{\mathbf{S}_k, T_k; k \in \mathbf{N}\}$  with finite-state space  $\mathcal{S}$  satisfies the conditions for being *Markov Renewal Process*, and the process  $\{\mathbf{X}(t), t \geq 0\}$  can be called as the *semi-Markov process* (SMP) associated with  $\{\mathbf{S}_k, T_k; k \in \mathbf{N}\}$  [26]. Moreover, since the general distributions for destination, speed, and pause time parameters are assumed to be *time-homogeneous* in the model proposed, for each pair  $(s, s') \in \mathcal{S} \times \mathcal{S}$ , the distribution of state holding time in state  $s$  before moving to state  $s'$ , given that the next state to be visited is  $s'$ , would be independent of  $k$ . Hence, based on the results provided in [26] and [27] for the theory of semi-Markov

TABLE I  
TRANSITION PROBABILITIES OF THE PROCESS  $\{\mathbf{S}_k, k \in \mathbf{N}\}$

Event	Transition	Probability	Condition*
$E_1$	$(c_i, 0) \rightarrow (c_i, c_j, z_r, 1)$	$\frac{\tau_{j i}}{1 - \tau_{i i}} \nu_{r i,j}$	$i \neq j$
$E_2$	$(c_i, c_j, z_r, 1) \rightarrow (c_{i+1}, c_j, z_r, 1)$	1	$j > i + 1$
	$(c_i, c_j, z_r, 1) \rightarrow (c_{i-1}, c_j, z_r, 1)$	1	$j < i - 1$
$E_3$	$(c_i, c_j, z_r, 1) \rightarrow (c_j, 0)$	1	$ i - j  = 1$

\*  $i, j = 0, \dots, n-1, r = 1 \dots m$

processes, the transitions of the process  $\mathbf{X}(t)$  from state  $s$  to state  $s'$  at the time instants  $T_k$  can be governed by the *discrete-time* Markov chain (DTMC)  $\{\mathbf{S}_k, k \in \mathbf{N}\}$  with finite-state space  $\mathcal{S}$  and transition probability matrix  $P = [p_{ss'}]$ , where  $p_{ss'} = \Pr\{\mathbf{S}_{k+1} = s' \mid \mathbf{S}_k = s\}$ , such that  $\sum_{s' \in \mathcal{S}} p_{ss'} = 1$  for all  $s \in \mathcal{S}$ . The process  $\{\mathbf{S}_k, k \in \mathbf{N}\}$  is also called *embedded DTMC* of SMP.

Consequently, if the DTMC  $\{\mathbf{S}_k, k \in \mathbf{N}\}$  satisfies the *ergodicity* conditions, and if the mean state holding times are finite, then the SMP  $\{\mathbf{X}(t), t \geq 0\}$  can be characterized at the long-run. Clearly, if long-run proportion of times spent at the states of the discrete state space  $\mathcal{S}$  are known, then by aggregating the states that has the same current cell component, that is,  $c_i$ , the long-run location distribution for the discretized region can be easily obtained. After this, by observing the limiting behavior of that discrete result as  $n \rightarrow \infty$  and  $m \rightarrow \infty$ , the continuous result can be derived. The same approach can be also used to obtain long-run speed distribution but in that case, the states with the same speed component, that is,  $z_r$ , must be aggregated. In the following section, we will at first generate the irreducible stochastic matrix  $P$  explicitly. Then, we will apply this approach to derive long-run location and speed distributions of continuous case.

### III. ANALYTICAL RESULTS FOR DISCRETIZED AND CONTINUOUS MOBILITY FORMULATIONS

In this section, we apply our solution framework with the ultimate aim of finding closed form expressions for the long-run location and speed distributions over  $R = [0, a]$ .

Now to describe the transition probabilities of the embedded DTMC  $\{\mathbf{S}_k, k \in \mathbf{N}\}$ , we first define:

$$\begin{aligned} \tau_{j|i} &= \Pr\{X_d \in c_j \mid X_s \in c_i\} \\ &= \int_{j \Delta x}^{(j+1) \Delta x} dx_d f_{X_d|X_s}(x_d \mid X_s \in c_i), \end{aligned} \quad (2)$$

for  $i, j = 0, \dots, n-1$ . Next, since  $V$  is allowed to be dependent on  $X_s$  and  $X_d$ , we define the probability mass function of  $V^*$  given  $X_s \in c_i$  and  $X_d \in c_j$ , that is, for a movement epoch that had started at  $c_i$  and destined to  $c_j$ , by

$$\begin{aligned} \nu_{r|i,j} &= \Pr\{V^* = z_r \mid X_s \in c_i, X_d \in c_j\} \\ &= \int_{(r-1)\Delta v}^{r\Delta v} f_{V|X_s, X_d}(v \mid X_s \in c_i, X_d \in c_j) dv, \end{aligned} \quad (3)$$

for  $r = 1, \dots, m$ .

Based on the events  $E_1$ ,  $E_2$ , and  $E_3$  that cause state changes, and  $\tau_{j|i}$  and  $\nu_{r|i,j}$  and given above, the possible transitions and the corresponding transition probabilities of the embedded DTMC can be grouped as in Table I.

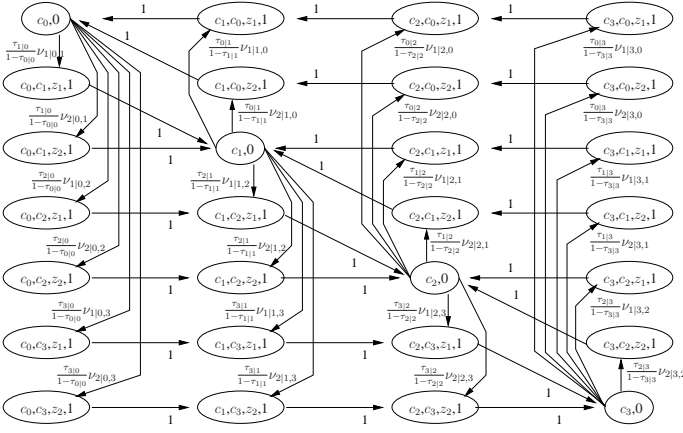


Fig. 2. State transition diagram for the process  $\{S_k, k \in \mathbb{N}\}$ , where  $n = 4$  and  $m = 2$ .

It should be noted from Table I that when  $E_1$  occurs, the mobile that is located at  $c_i$  jumps to moving mode in the current cell occupied. We enforced these transitions for the purpose of uniquely identifying moving and pausing terminals. In Fig. 2 we depicted the state transition diagram of the process  $\{S_k, k \in \mathbb{N}\}$  for a simple case where  $n=4$  and  $m=2$ .

Next, we formulate the transition probability matrix  $P$  of the process  $\{S_k, k \in \mathbb{N}\}$  in full generality. Clearly the structure of the matrix  $P$  depends on the order imposed on the states in  $\mathcal{S}$ . The ordering that we have decided on is  $\mathcal{S} = \{S_0, S_1, \dots, S_{n-1}\}$ , where each  $S_i$  has  $m(n-1) + 1$  states according to the following order:

$$\begin{aligned} \mathcal{S}_i = \{ & (c_i, c_0, z_1, 1), \dots, (c_i, c_0, z_m, 1), \dots, \\ & (c_i, c_{i-1}, z_1, 1), \dots, (c_i, c_{i-1}, z_m, 1), (c_i, 0), \\ & (c_i, c_{i+1}, z_1, 1), \dots, (c_i, c_{i+1}, z_m, 1), \dots, \\ & (c_i, c_{n-1}, z_1, 1), \dots, (c_i, c_{n-1}, z_m, 1) \}. \end{aligned} \quad (4)$$

Based on this ordering, the transition probability matrix  $P$  has the following discrete-time level-dependent quasi-birth-and-death process (QBD) form [28]:

$$P = \begin{bmatrix} A_1^{(0)} & A_0^{(0)} & & & \\ A_2^{(1)} & A_1^{(1)} & A_0^{(1)} & & \\ & \ddots & \ddots & \ddots & \\ & & A_2^{(n-2)} & A_1^{(n-2)} & A_0^{(n-2)} \\ & & & A_2^{(n-1)} & A_1^{(n-1)} \end{bmatrix} \quad (5)$$

where the matrices  $A_0^{(i)}$ ,  $A_1^{(i)}$ , and  $A_2^{(i)}$ ,  $i = 0, \dots, n-1$ , are  $(m(n-1) + 1) \times (m(n-1) + 1)$ , and defined as

$$\begin{aligned} A_0^{(i)} &= \begin{bmatrix} \mathbf{e}_0^{(i)} & & \\ & I_1 & \\ & & I_{m(n-i-2)} \end{bmatrix}, \quad A_2^{(i)} = \begin{bmatrix} I_{m(i-1)} & & \\ & I_1 & \\ & & \mathbf{e}_2^{(i)} \end{bmatrix}, \\ A_1^{(i)} &= \begin{bmatrix} B_0^{(i)} & \dots & B_{i-1}^{(i)} & B_{i+1}^{(i)} & \dots & B_{n-1}^{(i)} \end{bmatrix} \end{aligned} \quad (6)$$

where upper left block of  $A_1^{(i)}$  is a zero matrix of size  $m \times m i$ , and  $I_h$  denote the  $h \times h$  identity matrix for some positive

integer  $h$ . Moreover, the  $1 \times m$  row vector  $B_j^{(i)}$ , for  $i, j = 0, \dots, n-1$ , where  $i \neq j$ , and column vectors  $\mathbf{e}_0^{(i)}$ ,  $\mathbf{e}_2^{(i)}$  of respective sizes  $m(i+1) \times 1$  and  $m(n-i) \times 1$ , are respectively defined by

$$B_j^{(i)} = \frac{\tau_j |i|}{1 - \tau_j |i|} \nu_{m|i,j}, \quad \mathbf{e}_0^{(i)} = [0, \dots, 0, e_{m-1}]^T, \quad \mathbf{e}_2^{(i)} = [e_{m-1}, 0, \dots, 0]^T \quad (7)$$

where  $\nu_{m|i,j} = [\nu_{1|i,j}, \dots, \nu_{m|i,j}]$ , and  $e_h$  is the  $1 \times h$  vector of ones. The remaining blocks of the matrices  $A_0^{(i)}$ ,  $A_1^{(i)}$ , and  $A_2^{(i)}$  are zero matrices of sizes that can be easily derived from the dimensions of the other blocks.

Before we can proceed with the long-run analysis of the SMP  $\{X(t), t \geq 0\}$ , we must first find the steady-state distribution of the embedded DTMC  $\{S_k, k \in \mathbb{N}\}$  with the transition probability matrix  $P$  given in (5). Clearly, this distribution exists if and only if a steady-state distribution exists for  $X_s$ , and  $\{S_k, k \in \mathbb{N}\}$  satisfies the ergodicity conditions. Hence, we focus on these issues now.

Under the ‘‘mild’’ regularity conditions defined by Feller [25] on  $f_{X_d|X_s}(x_d|x_s)$ , there exists a steady-state distribution for  $X_s$  with pdf  $f_{X_s}(x_d)$ , which can be uniquely determined from the solution of the following integral equation

$$f_{X_s}(x_d) = \int_0^a f_{X_d|X_s}(x_d|x_s) f_{X_s}(x_s) dx_s \quad (8)$$

Observe that the integral equation given above is used to obtain the steady-state behavior of the discrete-time<sup>2</sup>, continuous-state Markov process  $\{X_s\}$ . From a different perspective, let  $T = [\tau_j |i|]$ , and let  $\varphi_i$ ,  $i = 0, \dots, n-1$  denote the probability of starting a movement epoch from cell  $c_i$  at the steady-state. Based on these notations, this integral equation is just the *analog* version of  $\varphi T = \varphi$  with  $\|\varphi\|_1 = 1$ , where  $\varphi = [\varphi_0, \dots, \varphi_{n-1}]$ . Clearly if the distribution of  $X_d$  is assumed to be independent from  $X_s$ , then the solution of the integral equation (8) would be simple. However, for other cases, deriving  $f_{X_s}(x_d)$  can be a very tedious task. We will return back this point later in Subsection III-B that concentrates on the mobility scenarios where choice of  $X_d$  is dependent on  $X_s$ .

Hence, if the pdf  $f_{X_s}(x_d)$  can be uniquely determined from the solution of (8), then  $\varphi_i$  will be given by

$$\varphi_i = \int_{i \Delta x}^{(i+1) \Delta x} dx_d f_{X_s}(x_d) \quad (9)$$

Next, we examine the ergodicity of  $\{S_k, k \in \mathbb{N}\}$ .

**Lemma 1:** If the pdf  $f_{X_s}(x_d)$  can be uniquely determined from the integral equation (8), and if  $\nu_r |i,j| > 0$ ,  $i, j = 0, \dots, n-1$  and  $r = 1, \dots, m$ , then the embedded DTMC  $\{S_k, k \in \mathbb{N}\}$  defined on state space  $\mathcal{S} = \{S_0, S_1, \dots, S_{n-1}\}$ , with transition probability matrix  $P$  defined as in (5), will be irreducible and aperiodic.

*Proof:* Please refer to [29]. ■

Thus, when the conditions of *ergodicity* for the DTMC  $\{S_k, k \in \mathbb{N}\}$  are satisfied, the steady-state distribution of it,

<sup>1</sup>Since  $X_d$  is the  $X_s$  of the next mobility epoch,  $X_s$  and  $X_d$  can be used interchangeably at the long-run.

<sup>2</sup>The stochastic process  $\{X_s\}$  changes its state at embedded time instants that represent the starting time of a new movement epoch.

which we denote by  $\pi_s$  for state  $s \in \mathcal{S}_i$ ,  $i = 0 \dots n-1$ , can be uniquely determined by solving the matrix equation

$$\pi P = \pi, \text{ with } \|\pi\|_1 = 1 \quad (10)$$

where  $\pi = [\pi_0, \pi_1, \dots, \pi_{n-1}]$ , and  $\pi_i$  is a (row) vector of size  $m(n-1) + 1$  whose elements are  $\pi_s$ ,  $\forall s \in \mathcal{S}_i$ , according to the order given by (4).  $\pi_i$  can be also called the solution vector for level  $i$ ,  $i = 0, \dots, n-1$ , as in [30].

Next, we examine the solution of the linear system given by (10). To the best of our knowledge, if there are no additional assumptions made on the properties of the matrix  $P$ , the most efficient direct computational procedure to find the steady-state distribution of finite-state level-dependent QBDs is presented in [28]. By using that procedure, one can obtain  $\pi$  numerically for some moderate values of  $n$  and  $m$ . However, as we made clear before, we are aimed at finding the limiting behavior of the long-run distributions for the discretized case as  $\{n, m\} \rightarrow \infty$ . Clearly this can only be done after deriving the location and speed distributions in closed form expressions. Therefore, we focused on an alternative direct approach and derived the following result.

**Lemma 2:** If the conditions given in Lemma 1 for the ergodicity of the DTMC  $\{\mathbf{S}_k, k \in \mathbf{N}\}$  are satisfied, then the solution vector  $\pi_i$  for level  $i$ ,  $i = 0, \dots, n-1$ , of the level-dependent QBD process given in (5), with the matrices  $A_0^{(i)}$ ,  $A_2^{(i)}$ , and  $A_1^{(i)}$  defined as in (6), is given by

$$\pi_i = [\pi_{i,0}, \dots, \pi_{i,i}, \dots, \pi_{i,n-1}]/N \quad (11)$$

where

$$\pi_{i,j} = \begin{cases} \sum_{\ell=i}^{n-1} \varphi_\ell \tau_{j|\ell} \nu_{m|\ell,j}, & \text{if } j < i \\ \varphi_i (1 - \tau_{i|i}), & \text{if } j = i \\ \sum_{\ell=0}^i \varphi_\ell \tau_{j|\ell} \nu_{m|\ell,j}, & \text{if } j > i \end{cases}, \quad (12)$$

where  $\nu_{m|i,j} = [\nu_{1|i,j}, \dots, \nu_{m|i,j}]$ , and  $N = \sum_{i=0}^{n-1} \|\pi_i\|_1$ .

*Proof:* Please refer to [29]. ■

To characterize the SMP  $\{\mathbf{X}(t), t \geq 0\}$  at the long-run, it remains to formulate the expected state holding times. For this purpose, let  $\bar{t}_s$  be the expected holding time in state  $s \in \mathcal{S}$ . Recall that in Section II, we decomposed the state space  $\mathcal{S}$  into two groups that represent moving (i.e.,  $q = 1$ ), and pausing (i.e.,  $q = 0$ ) terminals. Therefore, since  $R = [0, a]$  is discretized by cells of size  $\Delta x$ , the expected time that is going to be spent at a cell  $c_i$  by moving terminals is simply

$$\bar{t}_s = \frac{\Delta x}{z_r} \quad (13)$$

where  $s = (c_i, c_j, z_r, 1)$ ,  $i, j = 0, \dots, n-1$ , and  $r = 1, \dots, m$ , such that  $i \neq j$ . To formulate the mean time that is spent in a state of the form  $s = (c_i, 0)$ ,  $i = 0, \dots, n-1$ , we also define the following notation:

$$\begin{aligned} \bar{t}_s = E[T_{p_i}] &= E[T_p | X_s \in c_i] \\ &= \int_0^\infty \Pr\{T_p > t_p | X_s \in c_i\} dt_p \end{aligned} \quad (14)$$

Notice that the following equation

$$\sum_{s \in \mathcal{S}} \pi_s \bar{t}_s < \infty \quad (15)$$

is satisfied only if the minimum speed a mobile can attain is nonzero, and mean pause time spent at destinations are finite. Hence, if the mobility characterization parameters  $f_{V|X_s, X_d}$  and  $f_{T_p|X_d}$  are selected appropriately to satisfy these conditions, then the conditions given in [26] for the long-run characterization of SMPs are satisfied, and  $P_s$ , which corresponds to long-run proportion of time that the process is in state  $s$ , is simply

$$P_s = \frac{\pi_s \bar{t}_s}{\sum_{s' \in \mathcal{S}} \pi_{s'} \bar{t}_{s'}}, \quad \forall s \in \mathcal{S} \quad (16)$$

Finally, after aggregating the states that belong to the same level (i.e.,  $\mathcal{S}_i$ ,  $i = 0, \dots, n-1$ ) of the level-dependent QBD process  $\{\mathbf{S}_k, k \in \mathbf{N}\}$ , we obtained the following result for the long-run location distribution of the discretized one-dimensional regions.

**Lemma 3:** For the mobile terminal, whose mobility pattern is formulated according to the discretized version of the  $< f_{X_d|X_s}, f_{V|X_s, X_d}, f_{T_p|X_d} >$  mobility characterization, let  $p_i$ ,  $i = 0, \dots, n-1$ , denote the long-run proportion of time that terminal stays in cell  $c_i$ . If the conditions given in Lemma 1 holds, and if the equation (15) is satisfied, then

$$p_i = \frac{\varphi_i (1 - \tau_{i|i}) E[T_{p_i}] + k_i \Delta x}{\sum_{i=0}^{n-1} \varphi_i (1 - \tau_{i|i}) E[T_{p_i}] + \hat{D}_n \Delta x} \quad (17)$$

where

$$\begin{aligned} k_i &= \sum_{j=0}^{i-1} \sum_{\ell=i}^{n-1} \varphi_\ell \tau_{j|\ell} \sum_{r=1}^m \frac{1}{z_r} \nu_{r|\ell,j} \\ &+ \sum_{j=i+1}^{n-1} \sum_{\ell=0}^i \varphi_\ell \tau_{j|\ell} \sum_{r=1}^m \frac{1}{z_r} \nu_{r|\ell,j}, \end{aligned} \quad (18)$$

and

$$\hat{D}_n = \sum_{i=0}^{n-1} k_i \quad (19)$$

*Proof:* Please refer to [29]. ■

Next, we turn our attention to the limiting behavior of the discrete result derived in Lemma 3, and summarize our fundamental result for the long-run location distribution.

**Theorem 1:** For the mobile terminal, whose mobility pattern is characterized by  $< f_{X_d|X_s}, f_{V|X_s, X_d}, f_{T_p|X_d} >$ , let  $f_X(x)$ ,  $x \in [0, a]$ , denote the pdf of its location distribution at the long-run. If the pdf  $f_{X_s}(x_d)$  can be uniquely determined from the integral equation (8), and  $E[T_p | X_s = x_s] < \infty$ ,  $\forall x_s \in [0, a]$ , and  $f_{V|X_s, X_d} > 0$ ,  $\forall v \in [v_{\min}, v_{\max}]$ , and  $\forall x_s, x_d \in [0, a]$ , then

$$f_X(x) = \frac{f_{X_s}(x) E[T_p | X_s = x] + k_X(x)}{E[T_p | 0 \leq X_s \leq a] + \hat{D}} \quad (20)$$

where

$$k_X(x) = \int_0^x dx_d \int_x^a dx_s g_X(x_s, x_d) + \int_x^a dx_d \int_0^x dx_s g_X(x_s, x_d) \quad (21)$$

where

$$g_X(x_s, x_d) = f_{X_s}(x_s) f_{X_d|X_s}(x_d | x_s) E[\frac{1}{v} | X_s = x_s, X_d = x_d], \quad (22)$$

and

$$E[\frac{1}{V} | X_s = x_s, X_d = x_d] = \int_{v_{\min}}^{v_{\max}} dv \frac{1}{v} f_{V|X_s, X_d}(v | x_s, x_d), \quad (23)$$

and

$$\hat{D} = \int_0^a dx k_X(x) \quad (24)$$

*Proof:* Please refer to [29]. ■

It should be noted that if the distribution of  $V$  is independent from  $X_s$  and  $X_d$ , then the pdf  $f_V$  can be employed instead of  $f_{V|X_s, X_d}$  for mobility characterization, and  $k_X$  (21), and  $\hat{D}$  (24) simplifies to

$$k_X(x) = E[\frac{1}{V}] \int_0^x dx_d \int_x^a dx_s f_{X_s}(x_s) f_{X_d|X_s}(x_d | x_s) + E[\frac{1}{V}] \int_x^a dx_d \int_0^x dx_s f_{X_s}(x_s) f_{X_d|X_s}(x_d | x_s) \quad (25)$$

$$\hat{D} = E[\frac{1}{V}] \bar{D} \quad (26)$$

where

$$\bar{D} = \int_0^a dx_d \int_0^{x_d} dx_s (x_d - x_s) f_{X_s}(x_s) f_{X_d|X_s}(x_d | x_s) + \int_0^a dx_d \int_{x_d}^a dx_s (x_s - x_d) f_{X_s}(x_s) f_{X_d|X_s}(x_d | x_s) \quad (27)$$

Notice that  $\bar{D}$  is actually the average distance between the two points  $X_s$  and  $X_d$  drawn at random according to the pdfs  $f_{X_s}$ , and  $f_{X_d|X_s}$ , respectively. In addition, if  $X_d$  is also independent from  $X_s$ , then  $f_{X_d}$  can be used instead of  $f_{X_d|X_s}$ , and  $k_X$  (21) further simplifies to

$$k_X(x) = 2 E[\frac{1}{V}] \int_0^x dx_d \int_x^a dx_s f_{X_d}(x_s) f_{X_d}(x_d) \quad (28)$$

Having defined the long-run location distribution, we now concentrate on the long-run speed distribution. Clearly in order to achieve this, we need to aggregate the steady-state probabilities of the states in  $\mathcal{S}$  that has the same speed component, and take the limit of the resulting expression as  $\{\Delta x, \Delta v\} \rightarrow 0$ . Thus, for the mobile terminal whose movement behavior is characterized according to the triplet  $\langle f_{X_d|X_s}, f_{V|X_s, X_d}, f_{T_p|X_d} \rangle$ , let the continuous random variable  $\tilde{V}(t)$  defined on the state space  $\{0\} \cup \{v | v_{\min} \leq v \leq v_{\max}\}$  denote the speed of a mobile terminal at time  $t$ . Note that, since the mobile can be in pausing mode at some point in time,  $\tilde{V}(t)$  can also attain the zero value. Next, let  $\tilde{V}$  represent the random variable having the long-run distribution of  $V(t)$ , and denote its pdf by  $f_{\tilde{V}}$ . Finally, referring back to assumption  $A_2$ , denote the discrete random variable  $\tilde{V}^*$  defined on the state space  $\mathcal{S}_{\tilde{V}^*} = \{0\} \cup \mathcal{S}_{V^*} = \{z_0, z_1, z_2, \dots, z_m\}$ , where  $z_r = r \Delta v$ ,  $r = 0, 1, \dots, m$ , as the discrete approximation to the continuous random variable  $\tilde{V}$ .

Now, let  $\psi_r$  denote the long-run proportion of time that a mobile possesses speed  $z_r$ ,  $r = 0, 1, \dots, m$ . After aggregating the components of the vectors  $\pi_{i,j}$ ,  $i, j = 0, \dots, n-1$ , defined by (12) according to the states in  $\mathcal{S}$  that have the same speed component, and using the mean times that are going to be

spent in those states we get

$$\psi_r = \begin{cases} \left( \sum_{i=0}^{n-1} \varphi_i (1 - \tau_{i|i}) E[T_{p_i}] \right) / \hat{N}, & \text{if } r = 0 \\ \left( \sum_{i=0}^{n-1} \Delta x \left( \sum_{j=0}^{i-1} \sum_{\ell=i}^{n-1} \varphi_\ell \tau_{j|\ell} \frac{1}{z_r} \nu_{r|\ell, j} + \sum_{j=i+1}^{n-1} \sum_{\ell=0}^i \varphi_\ell \tau_{j|\ell} \frac{1}{z_r} \nu_{r|\ell, j} \right) \right) / \hat{N}, & \text{else} \end{cases} \quad (29)$$

where  $\hat{N} = \sum_{i=0}^{n-1} \varphi_i (1 - \tau_{i|i}) E[T_{p_i}] + \hat{D}_n \Delta x$ . Taking the limit of this discrete result as  $\{\Delta x, \Delta v\} \rightarrow 0$ , we reached to the following theorem.

**Theorem 2:** For the mobile terminal, whose mobility pattern is characterized by  $\langle f_{X_d|X_s}, f_{V|X_s, X_d}, f_{T_p|X_d} \rangle$ , if the conditions that are given in Theorem 1 for the parameters  $f_{X_d|X_s}$ ,  $E[T_p|X_s = x_s]$ , and  $f_{V|X_s, X_d}$  are satisfied, then

$$f_{\tilde{V}}(\tilde{v}) = \begin{cases} \frac{E[T_p|0 \leq X_s \leq a] \delta(\tilde{v})}{E[T_p|0 \leq X_s \leq a] + \bar{D}}, & \text{if } \tilde{v} = 0 \\ \frac{\int_0^a dx k_{\tilde{V}}(x, \tilde{v})}{E[T_p|0 \leq X_s \leq a] + \bar{D}}, & \text{if } \tilde{v} \in [v_{\min}, v_{\max}] \end{cases}, \quad (30)$$

and

$$E[\tilde{V}] = \frac{\bar{D}}{E[T_p|0 \leq X_s \leq a] + \bar{D}} \quad (31)$$

where

$$k_{\tilde{V}}(x, \tilde{v}) = \int_0^x dx_d \int_x^a dx_s g_{\tilde{V}}(x_s, x_d, \tilde{v}) + \int_x^a dx_d \int_0^x dx_s g_{\tilde{V}}(x_s, x_d, \tilde{v}), \quad (32)$$

where

$$g_{\tilde{V}}(x_s, x_d, \tilde{v}) = f_{X_s}(x_s) f_{X_d|X_s}(x_d | x_s) \frac{1}{\tilde{v}} f_{V|X_s, X_d}(\tilde{v} | x_s, x_d) \quad (33)$$

*Proof:* Please refer to [29]. ■

It should be noted that, if the distributions of  $X_d$  and  $T_p$  are independent from  $X_s$ , and if distribution of  $V$  is also independent of  $X_s$  and  $X_d$ , then the mobility characterization can be done by the triplet  $\langle f_{X_d}, f_V, f_{T_p} \rangle$ , and the formulation of  $f_{\tilde{V}}(v)$  and  $E[\tilde{V}]$  for this simplified mobility formulation will match to the results that are derived in [8] for a class of mobility models where speed is selected independently from the distance that is going to be traveled.

Finally, from the results presented by Theorems 1 and 2, it is clear that the dependency of  $X_d$  on  $X_s$  makes the fundamental difference. Therefore, in the following two subsections, we will at first concentrate on some example scenarios that uses  $f_{X_d}$  (i.e. distribution of  $X_d$  is independent from  $X_s$ ) for mobility characterization. Then, we will proceed to more complicated scenarios by employing the stochastic density kernel  $f_{X_d|X_s}$  for mobility formulation.

#### A. Variants of mobility characterizations done by $f_{X_d}$

**Example 1:** The random waypoint model [3] represents the simplest nontrivial case of our generalized modeling approach, and can be characterized according to the triplet

$< f_{X_d}, f_V, f_{T_p|X_d} >$ , where the parameters are defined by

$$f_{X_d}(x_d) = \begin{cases} \frac{1}{a}, & \text{if } 0 \leq x_d \leq a \\ 0, & \text{otherwise} \end{cases}, \quad (34)$$

$$f_V(v) = \begin{cases} \frac{1}{v_{\max} - v_{\min}}, & \text{if } v_{\min} \leq v \leq v_{\max} \\ 0, & \text{otherwise} \end{cases} \quad (35)$$

and

$$f_{T_p|X_d}(t_p|x_d) = \begin{cases} h(t_p), & \text{if } t_p \geq 0 \\ 0, & \text{otherwise} \end{cases} \quad (36)$$

where  $h(t_p)$  is the pdf of the random variable  $T_p$ , which is independent from the location of the destination. Denoting the average time spent at the destinations by  $E[T_p]$  (i.e.,  $E[T_p] = \int_0^\infty t_p h(t_p) dt_p$ ), observing

$$E\left[\frac{1}{V}\right] = \frac{\ln\left(\frac{v_{\max}}{v_{\min}}\right)}{(v_{\max} - v_{\min})}, \quad (37)$$

and using Theorems 1 and 2, we obtained the following for the pdf of the long-run location distribution and the expected value of speed at the long-run:

$$f_X(x) = \frac{\frac{1}{a}E[T_p] + \frac{2x(a-x)}{a^2}E\left[\frac{1}{V}\right]}{E[T_p] + \frac{a}{3}E\left[\frac{1}{V}\right]}, \quad (38)$$

$$E[\tilde{V}] = \frac{a/3}{E[T_p] + \frac{a}{3}E\left[\frac{1}{V}\right]} \quad (39)$$

We note that if the speed choice for each movement epoch is deterministic with a parameter  $v$ , then  $E\left[\frac{1}{V}\right]$  must be substituted with  $\frac{1}{v}$ . In addition, the analytical work presented in [21], considers two different limited variations of the one-dimensional result we derived for location distributions. At first, they concentrate on the case where  $E[T_p] = 0$ . Next, they extend their analysis, and provide the location distribution for the scenario where pause time is nonzero, and speed is deterministic (i.e., constant speed). For these two cases, if we make the appropriate changes in the formation of  $f_X$  given in (38) (i.e.,  $E[T_p] = 0$  for first case, and  $E\left[\frac{1}{V}\right] = \frac{1}{v}$  for the other case), then the results will match the pdfs presented in [21].

**Example 2:** In the random waypoint mobility model we analyzed by Example 1,  $V$  is assumed to be independent from  $|X_s - X_d|$ , that is, the distance traveled during a movement epoch. However, in most of the realistic scenarios,  $V$  tends to increase as  $|X_s - X_d|$  does. Thus, for this example, we make an improvement on the random waypoint model by proposing a  $f_{V|X_s, X_d}$  that provides the opportunity to determine  $V$  proportional to the random variable  $D = |X_s - X_d|$  with high probability.

Now, considered a truncated normal distribution [31] for  $V$  according to the pdf given by

$$f_{V|X_s, X_d}(v|x_s, x_d) = \frac{Z\left(\frac{v - \mu(x_s, x_d)}{\sigma}\right)}{\sigma\left(\Phi\left(\frac{v_{\max} - \mu(x_s, x_d)}{\sigma}\right) - \Phi\left(\frac{v_{\min} - \mu(x_s, x_d)}{\sigma}\right)\right)} \quad (40)$$

for  $v_{\min} \leq v \leq v_{\max}$  where  $\sigma > 0$ , and

$$\mu(x_s, x_d) = v_{\min} + \frac{(v_{\max} - v_{\min})}{a}|x_s - x_d| \quad (41)$$

$Z$  and  $\Phi$  are the probability density and cumulative distribution functions for the normal distribution [31].

Hence, we reached to the following results for this improved case:

$$f_X(x) = \frac{\frac{1}{a}E[T_p] + k_X(x)}{E[T_p] + \hat{D}}, \quad (42)$$

$$E[\tilde{V}] = \frac{a/3}{E[T_p] + \hat{D}} \quad (43)$$

where

$$k_X(x) = \frac{2}{a^2} \int_0^x dx_d \int_x^a dx_s \int_{v_{\min}}^{v_{\max}} dv \frac{1}{v} f_{V|X_s, X_d}(v|x_s, x_d) \quad (44)$$

where  $f_{V|X_s, X_d}$  is defined by (40).

Clearly, because of the complicatedness of  $f_{V|X_s, X_d}$ ,  $k_X(x)$  can only be evaluated numerically for a given  $x \in [0, a]$ , and also  $\hat{D}$ . However, for the extreme case  $\sigma \rightarrow 0$ , we have

$$f_{V|X_s, X_d}(v|x_s, x_d) = \delta(v - \mu(x_s, x_d)) \quad (45)$$

From a different point of view, for the limiting case where  $\sigma \rightarrow 0$ ,  $V$  will be linearly dependent to  $|X_s - X_d|$  with respect to the following transformation:

$$V = v_{\min} + \frac{(v_{\max} - v_{\min})}{a}|X_s - X_d| \quad (46)$$

Thus, the  $k_X(x)$  given by (44) simplifies to

$$k_X(x) = 2\left(\ln((v_{\max}(a-x) + xv_{\min})/a)(x(v_{\max} - v_{\min}) - v_{\max}a) + \ln((x(v_{\max} - v_{\min}) + v_{\min}a)/a)(x(v_{\min} - v_{\max}) - av_{\min}) + av_{\min} \ln(v_{\min}) + av_{\max} \ln(v_{\max})\right)/(a(v_{\max} - v_{\min})^2), \quad (47)$$

and  $\hat{D}$  will be given by

$$\hat{D} = \frac{a(v_{\max}^2 - v_{\min}^2 - 2v_{\min}v_{\max} \ln(\frac{v_{\max}}{v_{\min}}))}{(v_{\max} - v_{\min})^3} \quad (48)$$

Now, after substituting the  $\hat{D}$  given above by (48) to the equation for  $E[\tilde{V}]$  (43), a comparison of that  $E[\tilde{V}]$  with the one defined by (39) in Example 1 reveals that since  $\hat{D}$  (48) is less than  $(a/3)\frac{\ln(v_{\max}/v_{\min})}{v_{\max} - v_{\min}}$  (i.e.,  $\hat{D}$  in Example 1) for all  $v_{\max} > v_{\min} > 0$ , the  $E[\tilde{V}]$  obtained for the uniformly distributed  $V$  is always smaller than its counterpart for the  $V$  that is defined by (46). This is consistent with the intuitive expectations because when  $V = v_{\min} + ((v_{\max} - v_{\min})/a)|x_s - x_d|$ , the possibility of moving long distances with low speeds becomes zero. On the other hand, for the original random waypoint mobility model, since  $V$  is not directly proportional to  $D$ , lower speeds might be selected for longer distances and as a result, expected value of the long-run speed decreases. It should be also noted that, as  $E[T_p] \rightarrow 0$  and  $v_{\max} \rightarrow v_{\min}$ ,  $E[\tilde{V}]$  converges to  $v_{\min}$  for both choices of  $V$ , which is also expected because it corresponds to the scenario where mobile travels with fixed speed  $v_{\min}$  at all times without pausing at any destination.

The other extreme case of interest for this example is  $\sigma \rightarrow \infty$ , which simplifies to the scenario where  $V$  is uniformly distributed in  $[v_{\min}, v_{\max}]$ . Therefore, we conjecture that, if  $f_{V|X_s, X_d}$  is defined according to (40), the lower bound for  $E[\tilde{V}]$  is given by (39) in Example 1, and the upper bound for it is given by (43) with the  $\hat{D}$  defined as in (48). Obviously, the difference between these bounds decreases as  $E[T_p] \rightarrow \infty$ , or  $v_{\min} \rightarrow v_{\max}$ .

### B. Variants of mobility characterizations done by $f_{X_d|X_s}$

*Example 3:* As a basic example of a scheme where distribution of  $X_d$  is dependent on  $X_s$ , consider a scenario where the closed region  $R = [0, a]$  is partitioned into two subregions  $R_1 = [0, c)$  and  $R_2 = [c, a)$  such that  $0 < c < a$ . In this setting, when the starting point  $X_s \in R_1$ , the destination point  $X_d$  for that epoch will be distributed uniformly over either  $R_2$  or  $R_1$  with respective probabilities  $\alpha$  and  $(1 - \alpha)$  where  $0 < \alpha < 1$ . Similarly, if  $X_s \in R_2$ , then  $X_d$  will be distributed uniformly over either  $R_1$  or  $R_2$  with probabilities  $\beta$  and  $(1 - \beta)$ , respectively where  $0 < \beta < 1$ . Hence, the stochastic density kernel  $f_{X_d|X_s}$  will be formulated by

$$f_{X_d|X_s}(x_d|x_s) = \begin{cases} \frac{1-\alpha}{c}, & \text{if } x_s \in [0, c) \text{ and } x_d \in [0, c) \\ \frac{\alpha}{a-c}, & \text{if } x_s \in [0, c) \text{ and } x_d \in [c, a) \\ \frac{\beta}{c}, & \text{if } x_s \in [c, a) \text{ and } x_d \in [0, c) \\ \frac{1-\beta}{a-c}, & \text{if } x_s \in [c, a) \text{ and } x_d \in [c, a) \\ 0, & \text{otherwise} \end{cases} \quad (49)$$

where  $0 < \alpha < 1$  and  $0 < \beta < 1$ .

Now, let  $X_{d_k}$  denote the  $X_d$  (i.e. destination point) of the  $k$ th movement epoch. Then, based on the definition of  $f_{X_d|X_s}$  given above (49), we can construct the DTMC  $\{X_{d_k}, k \in \mathbf{N}\}$  with states that represent the subregions  $R_1, R_2$ , and a transition probability matrix  $A$  given by

$$A = \begin{matrix} R_1 \\ R_2 \end{matrix} \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix} \quad (50)$$

Obviously, the DTMC  $\{X_{d_k}, k \in \mathbf{N}\}$  governs the decisions of  $X_d$  at consecutive movement epochs, and in order to solve  $f_{X_s}(x_d)$  uniquely from integral equation (8), it must satisfy the conditions of ergodicity at first. Therefore,  $\alpha$  or  $\beta$  can not be equal to 0 or 1, which is already required in the formulation of  $f_{X_d|X_s}$ .

Hence, by applying the integral equation defined in (8), we can derive the  $f_{X_s}(x_d)$  for this example as follows:

$$f_{X_s}(x_d) = \begin{cases} \frac{1-\alpha}{c} \int_0^c f_{X_s}(x_s) dx_s \\ + \frac{\beta}{a-c} \int_c^a f_{X_s}(x_s) dx_s, & \text{if } x_d \in [0, c) \\ \frac{\alpha}{a-c} \int_0^c f_{X_s}(x_s) dx_s \\ + \frac{1-\beta}{a-c} \int_c^a f_{X_s}(x_s) dx_s, & \text{if } x_d \in [c, a) \end{cases} \quad (51)$$

which implies

$$f_{X_s}(x_d) = \begin{cases} k_1, & \text{if } x_d \in [0, c) \\ k_2, & \text{if } x_d \in [c, a) \end{cases} \quad (52)$$

for some constants  $k_1$  and  $k_2 \in \mathbf{R}$ .

Now, let  $\pi_A$  denote the steady state distribution of the DTMC  $\{X_{d_k}, k \in \mathbf{N}\}$  with transition probability  $A$ . Observe that  $\pi_A = [\frac{\beta}{(\alpha+\beta)}, \frac{\alpha}{(\alpha+\beta)}]$ , and since  $\{X_{d_k}, k \in \mathbf{N}\}$  determines the subregion  $X_d$  is located, we get

$$\int_0^c f_{X_s}(x_d) dx_d = \frac{\beta}{(\alpha+\beta)}, \int_c^a f_{X_s}(x_d) dx_d = \frac{\alpha}{(\alpha+\beta)} \quad (53)$$

which concludes

$$f_{X_s}(x_d) = \begin{cases} \frac{\beta}{(\alpha+\beta)} \frac{1}{c}, & \text{if } x_d \in [0, c) \\ \frac{\alpha}{(\alpha+\beta)} \frac{1}{(a-c)}, & \text{if } x_d \in [c, a) \end{cases} \quad (54)$$

Next, we focus on a more generic form for this scenario. Consider a partitioning of the region  $R = [0, a]$  into  $M$  subregions  $R_i = [a_i, a_{i+1})$ ,  $i = 1, \dots, M$  such that  $a_{i+1} > a_i$  with  $a_1 = 0$ ,  $a_{M+1} = a$ , and let the stochastic density kernel be defined by

$$f_{X_d|X_s}(x_d|x_s) = \begin{cases} \frac{A_{i,j}}{a_{j+1}-a_j}, & \text{if } x_s \in R_i, \text{ and } x_d \in R_j, \\ i, j = 1, \dots, M \\ 0, & \text{otherwise} \end{cases} \quad (55)$$

where  $A_{i,j}$  denote the probability of selecting  $X_d$  uniformly in subregion  $R_j$  given that  $X_s$  is located in subregion  $R_i$ .

Similar to the discussions for the solution of the integral equation given by (51), since the function  $f_{X_d|X_s}(x_d|x_s)$  is independent from  $x_d$  in all of the different subregions for  $x_s$ ,  $f_{X_s}(x_d)$  will be equal to a constant value in all of the subregions  $R_i$ ,  $i = 1, \dots, M$ , as in (52). Therefore, if the DTMC  $\{X_{d_k}, k \in \mathbf{N}\}$  with the  $M \times M$  transition probability matrix  $A = [A_{i,j}]$  is irreducible and aperiodic, then the stationary pdf of the destination points is given by

$$f_{X_s}(x_d) = \begin{cases} \frac{\pi_{A_i}}{a_{i+1}-a_i}, & \text{if } x_d \in R_i, i = 1, \dots, M \\ 0, & \text{otherwise} \end{cases} \quad (56)$$

where  $\pi_A = [\pi_{A_1}, \dots, \pi_{A_M}]$  is the solution of the linear system  $\pi_A A = \pi_A$ ,  $\|\pi_A\|_1 = 1$ .

As an application of this scenario, we focused on the one-dimensional version of the random direction model described in [32]. In this model, nodes are restricted to move between the destinations that are located at the  $\varepsilon$  neighborhood of boundaries. After reaching the destination, mobile pauses for a specified amount of time, and travels to a new destination, which is also located at the  $\varepsilon$  neighborhood of boundaries. Similar to the random waypoint mobility model, for each movement epoch,  $V$  is selected independently from  $|X_s - X_d|$ .

Now, in order to capture this model with the  $f_{X_d|X_s}$  defined by (55) on a one-dimensional topology, we have to set  $M = 3$ , and divide  $R$  into subregions  $R_1 = [0, \varepsilon)$ ,  $R_2 = [\varepsilon, a - \varepsilon)$ , and  $R_3 = [a - \varepsilon, a)$ . Since, the stochastic matrix  $A$  must be irreducible and aperiodic, we define it by

$$A = \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \begin{bmatrix} 0 & \epsilon & 1-\epsilon \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 1-\epsilon & \epsilon & 0 \end{bmatrix} \quad (57)$$

where  $0 < \epsilon < 1$ . Obviously, since  $\epsilon$  cannot be equal to 0, mobile terminals may select destination points located at  $R_2$ . However, as  $\epsilon \rightarrow 0$ , the possibility of this case diminishes, and we reach to desired scenario.

Hence, after obtaining the  $f_{X_s}(x_d)$  from (56) for a nonzero  $\epsilon$ , applying Theorem 1, and finally, by taking the limit of the result as  $\epsilon \rightarrow 0$ , we derived the following for the long-run location distribution of this mobility model:

$$f_X(x) = \begin{cases} \frac{E[T_p]/(2\varepsilon) + E[1/V]x/\varepsilon}{E[T_p] + \bar{D}}, & \text{if } x \in [0, \varepsilon) \\ \frac{E[1/V]}{E[T_p] + \bar{D}}, & \text{if } x \in [\varepsilon, a - \varepsilon) \\ \frac{E[T_p]/(2\varepsilon) + E[1/V](a-x)/\varepsilon}{E[T_p] + \bar{D}}, & \text{if } x \in [a - \varepsilon, a) \end{cases} \quad (58)$$

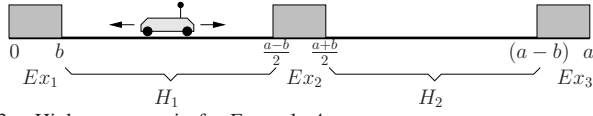


Fig. 3. Highway scenario for Example 4.

where  $\hat{D} = E[1/V](a - \varepsilon)$ , and  $E[T_p]$  is the expected pause time spent at the destinations. Notice that,  $f_X(x)$  converges to  $\frac{1}{a}$  as  $E[T_p] \rightarrow 0$  and  $\varepsilon \rightarrow 0$ .

Before proceeding to a more sophisticated scenario, we would like to emphasize an important issue about the usage of the stochastic density kernel  $f_{X_d|X_s}(x_d|x_s)$  for mobility characterization. Now observe that  $f_{X_d|X_s}$  provides a mechanism to accumulate the consecutive choices of destinations to subregions inside  $R$ . The transitions between the subregions can be also controlled by the transition probability matrix  $A$  we defined above. However,  $f_{X_d|X_s}$  can not be employed in controlling the direction of the mobile terminal at consecutive movement epochs. For example, on the region  $R = [0, a]$ , our formulation can not be used to capture a case where mobile selects the destinations towards the point  $a$ , with higher probability for each movement epoch. In order to have a probabilistic mechanism to control the direction, we must extend the mobility model with an underlying modulating Markov chain that controls direction by making transitions at the embedded times at which a new movement epoch starts. This is doable for the discretized version of the mobility formulation. However, it will never end up with tractable closed form expressions like the ones we presented by Lemmas 2 and 3.

**Example 4:** Consider the partitioning of the region  $R = [0, a]$  shown in Fig. 3 where mobile terminals are expected to move between destinations located in the subregions  $Ex_i$ ,  $i = 1, 2, 3$ , without pausing at the subregions  $H_1$  and  $H_2$ .

From a practical point of view, this partitioning can be considered as a highway scenario where  $Ex_i$  and  $H_i$  represent exit areas and highway segments, respectively. The exit areas can be also considered as hotspots where mobile accumulate with higher probability. Hence, for the purpose of using our  $\langle f_{X_d|X_s}, f_{V|X_s, X_d}, f_{T_p|X_d} \rangle$  mobility characterization approach to capture a highway scenario that is composed of movement epochs between exit areas or hotspots, suppose that if  $X_s \in Ex_i$ , then  $X_d$  will be uniformly distributed either over  $Ex_j$ , for  $j \neq i$ , or over  $H_j$ ,  $j = 1, 2$ , with respective probabilities  $\alpha$  and  $1/2 - \alpha$  where  $0 < \alpha < 1/2$ . Notice that, as  $\alpha \rightarrow 1/2$ , the possibility of a movement epoch to start from a highway segment, or to pause at somewhere on a highway segment becomes negligible. Furthermore, assume that if  $X_s \in H_i$ , then  $X_d$  will be uniformly distributed over either  $Ex_i$  or  $Ex_{i+1}$  with equal probabilities. Thus, the stochastic density kernel  $f_{X_d|X_s}$  will be given by

$$f_{X_d|X_s}(x_d|x_s) = \begin{cases} \frac{\alpha}{b}, & \text{if } x_s \in Ex_i \text{ and } x_d \in Ex_j, \\ & i \neq j \\ \frac{(1/2) - \alpha}{(a-3b)/2}, & \text{if } x_s \in Ex_i \text{ and } x_d \in H_j \\ \frac{1/2}{b}, & \text{if } x_s \in H_i \text{ and } x_d \in Ex_j, \\ & j = i, i+1 \\ 0, & \text{otherwise} \end{cases} \quad (59)$$

where  $0 < \alpha < 1/2$

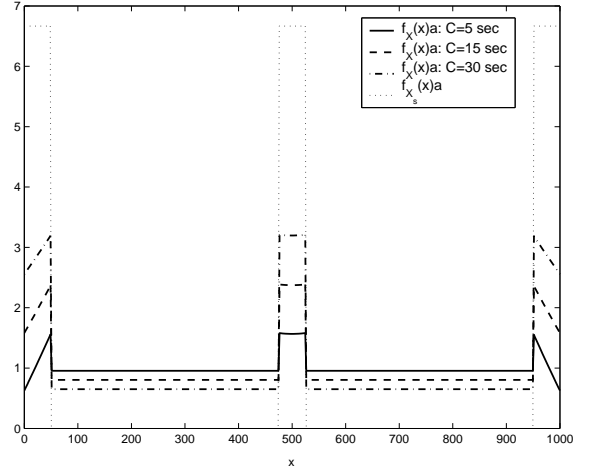


Fig. 4. Comparison of  $f_X$  and  $f_{X_s}$  as  $\alpha \rightarrow 1/2$  for Example 4. ( $a = 1000$  m,  $b = 50$  m,  $v_{\min} = 1$  m/s,  $v_{\max} = 20$  m/s,  $C \in \{5, 10, 15\}$  sec)

Based on this definition of  $f_{X_d|X_s}$ , the transition probability matrix  $A$  corresponding the DTMC  $\{X_{d_k}, k \in \mathbb{N}\}$  is

$$A = \begin{matrix} & \begin{matrix} Ex_1 \\ H_1 \\ Ex_2 \\ H_2 \\ Ex_3 \end{matrix} \end{matrix} \begin{bmatrix} 0 & \frac{1}{2} - \alpha & \alpha & \frac{1}{2} - \alpha & \alpha \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \alpha & \frac{1}{2} - \alpha & 0 & \frac{1}{2} - \alpha & \alpha \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \alpha & \frac{1}{2} - \alpha & \alpha & \frac{1}{2} - \alpha & 0 \end{bmatrix} \quad (60)$$

Observe that,  $A$  satisfies the conditions of ergodicity if and only if  $\alpha \neq 1/2$ , which is also required by the definition of  $f_{X_d|X_s}$ .

Hence, by applying the result given by equation (56) for the  $f_{X_d|X_s}$  of the form (55), we get

$$f_{X_s}(x_d) = \begin{cases} \frac{1+2\alpha}{8(1-\alpha^2)b}, & \text{if } x_d \in Ex_i, i = 1, 3 \\ \frac{1}{4(1-\alpha^2)b}, & \text{if } x_d \in Ex_2 \\ \frac{1-2\alpha}{2(1-\alpha)(a-3b)}, & \text{if } x_d \in H_j, j = 1, 2 \end{cases} \quad (61)$$

Furthermore, since we want the terminals to pause at only exit areas, we decided on the following function for the expected pause times at the destinations

$$E[T_p|X_s] = \begin{cases} C, & \text{if } X_s \in Ex_i, i = 1, 2, 3 \\ 0, & \text{otherwise} \end{cases} \quad (62)$$

where  $C$  is a constant  $> 0$ . In addition, we assume that  $V$  is defined by (46).

Based to the mobility characterization parameters we described, we generated  $k_X$  and  $\hat{D}$  defined in Theorem 1 by the dividing the ranges of the double integration operations confidently to the subregions defined above. After this, we derived the limiting expressions of them as  $\alpha \rightarrow 1/2$ , and finally we obtained the pdf  $f_X$  for that limiting case (i.e., the case where destinations are only selected at the exit areas). Since  $V$  is dependent on  $D$ , the final form of  $f_X$  is not simple enough to fully present here. However, plots of  $f_X$  for different cases, and a graphical comparison of it with  $f_{X_s}$  can be found in Fig. 4.

From Fig. 4, first observe that  $f_X$  and  $f_{X_s}$  are substantially different. This is expected because, during moving mobile terminal passes through highway segments and although they don't pause at highways, the proportion of time spent at

highways locations increases as they move between exit areas. Furthermore, as the expected value of pause times at the exit areas (i.e.  $C$ ) increases, the value of  $f_X$  at the highway segments decreases because they spend more time on the exit areas at the long-run. In addition, this example also shows that a performance analysis study that makes assumptions about the location distribution can not ignore the times spent on the highways that connect hotspots, or the subregions where mobile terminals accumulate with higher probability.

### C. Modeling Acceleration

The obvious unrealistic characteristic of the movement behavior generated by our generalized approach of mobility modeling is that at the beginning of a movement epoch that had started at  $X_s$  and destined to  $X_d$ , the instantaneous speed, that is, speed at any instant of time, of a mobile terminal jumps from 0 to  $V$  abruptly implying an acceleration that is  $\infty$  in magnitude. In addition, when mobile reaches to the destination, it decreases from  $V$  to 0 with a deceleration that is also  $\infty$  in magnitude. However, in realistic situations, the magnitudes of acceleration and deceleration are finite, and a mobile terminal cannot immediately increase its instantaneous speed from 0 to  $V$  at the point  $X_s$ , and also immediately drop it from  $V$  to 0 at the point  $X_d$ . Clearly other random walk or random waypoint like mobility models that we have mentioned in Section I also possesses this unrealistic characteristic.

Now in order to remove this unrealistic movement behavior from our mobility formulation, assume that for each movement epoch, a mobile terminal increases its speed from 0 to  $V$  *uniformly* with an acceleration that has *constant* magnitude, travels at speed  $V$  for a distance, and when it gets close to destination, it decreases its speed from  $V$  to 0 uniformly with an deceleration that is also *constant* in magnitude. Let  $\phi_{acc}$  and  $\phi_{dec}$  denote the magnitudes of acceleration and deceleration, respectively. Before we proceed further in the analysis, we assume that the distance between the point  $X_d$  and the location at which mobile starts slowing down must be exactly equal to the distance required to decrease speed from  $V$  to 0 with a deceleration that is equal to  $\phi_{dec}$  in magnitude (i.e., a symmetric environment). In addition, in the rest of this subsection, since terminals accelerate to and from speed  $V$ , which is drawn randomly from  $[v_{min}, v_{max}]$ , the random variable  $V$  will be also called as “target speed”. Hence, let  $f_\phi(X_s, X_d, V, \phi_{acc}, \phi_{dec}, X)$  denote the speed of the mobile terminal at the point  $X$  for the movement epoch between  $X_s$  and  $X_d$  with target speed  $V$ , constant acceleration  $\phi_{acc}$ , and constant deceleration  $\phi_{dec}$ . Notice that acceleration becomes 0 when the terminal reaches target speed  $V$ . However, for some exceptional cases, the absolute distance between  $X_s$  and  $X_d$  can be so small that the mobile might be forced to decelerate before reaching to target speed  $V$ . In order to illustrate these characteristics, in Fig. 5 and Fig. 6, we focused on a single movement epoch between the points  $X_s$  and  $X_d$  where  $X_d > X_s$ , and plotted the instantaneous speed of a terminal versus its location (i.e.  $X$ ). Observe that when destination  $X_d$  is too close to  $X_s$ , mobile terminal cannot reach instantaneous speed  $V$ , which is selected as the speed of the movement interval between  $X_s$  and  $X_d$ , and has to decelerate after an acceleration

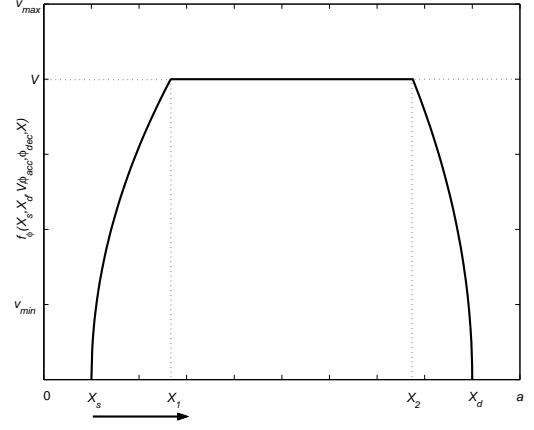


Fig. 5. Mobile reaches target speed  $V$ .

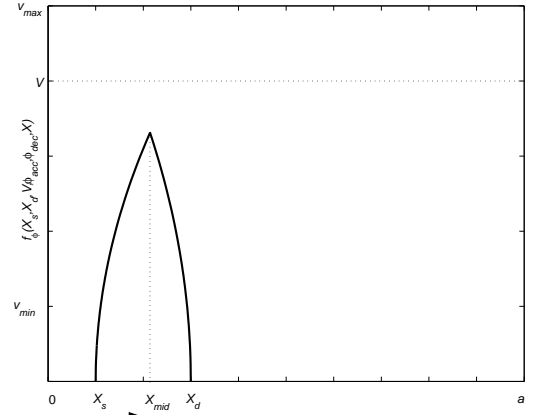


Fig. 6. Mobile slows down before reaching target speed  $V$ .

period. More formally, for the case where  $X_d > X_s$ , let  $X_1 = X_s + \frac{V^2}{2\phi_{acc}}$ , and  $X_2 = X_d - \frac{V^2}{2\phi_{dec}}$ . Hence, if  $X_1 \leq X_2$ , then

$$f_\phi(X_s, X_d, V, \phi_{acc}, \phi_{dec}, X) = \begin{cases} \sqrt{2\phi_{acc}(X - X_s)}, & X \in (X_s, X_1] \\ V, & X \in (X_1, X_2] \\ \sqrt{V^2 - 2\phi_{dec}(X - X_2)}, & X \in (X_2, X_d] \end{cases} \quad (63)$$

On the other hand, if  $X_1 > X_2$  (i.e., mobile must slow down before reaching speed  $V$ ), then

$$f_\phi(X_s, X_d, V, \phi_{acc}, \phi_{dec}, X) = \begin{cases} \sqrt{2\phi_{acc}(X - X_s)}, & X \in (X_s, X_{mid}] \\ \sqrt{2\phi_{dec}(X_d - X)}, & X \in (X_{mid}, X_d] \end{cases} \quad (64)$$

where  $X_{mid} = X_s + \frac{\phi_{dec}(X_d - X_s)}{\phi_{dec} + \phi_{acc}}$ .

For the other case where  $X_d < X_s$ , let  $X_1 = X_s - \frac{V^2}{2\phi_{acc}}$ , and  $X_2 = X_d + \frac{V^2}{2\phi_{dec}}$ . Hence, if  $X_1 \geq X_2$ , then

$$f_\phi(X_s, X_d, V, \phi_{acc}, \phi_{dec}, X) = \begin{cases} \sqrt{2\phi_{acc}(X_s - X)}, & X \in (X_1, X_s] \\ V, & X \in (X_2, X_1] \\ \sqrt{V^2 + 2\phi_{dec}(X - X_2)}, & X \in (X_d, X_2] \end{cases} \quad (65)$$

However, if  $X_1 < X_2$  (i.e., exceptional case), then

$$f_\phi(X_s, X_d, V, \phi_{acc}, \phi_{dec}, X) = \begin{cases} \sqrt{2\phi_{acc}(X_s - X)}, & X \in (X_{mid}, X_s] \\ \sqrt{2\phi_{dec}(X - X_d)}, & X \in (X_d, X_{mid}] \end{cases} \quad (66)$$

where  $X_{mid} = X_s - \frac{\phi_{dec}(X_s - X_d)}{\phi_{dec} + \phi_{acc}}$ .

We note that if  $\phi_{acc} = \infty$  and  $\phi_{dec} = \infty$ , then  $X_1 = X_s$  and  $X_2 = X_d$  for all of the cases we defined above, and consequently  $f_\phi(X_s, X_d, V, \phi_{acc}, \phi_{dec}, X) = V$  at all points between  $X_s$  and  $X_d$ .

It is now apparent from these formulations that in order to capture acceleration-deceleration characteristics of vehicles, mobility formulation must keep the information about the starting point (i.e.,  $X_s$ ) of each movement epoch that is destined to the point  $X_d$ . Since we employ the stochastic density kernel  $f_{X_d|X_s}$  in mobility characterization, this requirement has already been satisfied.

Now in order to formulate the long-run location and speed distributions according to the acceleration and deceleration parameters, we first need to extend the results given for the discretized mobility formulation. Returning back to Lemma 2, observe from the formulation of  $\pi_{i,j}$  (12) (i.e., the probability of being in cell  $c_i$  and moving towards cell  $c_j$  at the steady-state) that the steady-state probability of being at  $c_i$  for a movement epoch that had started at  $c_\ell$  and destined to  $c_j$  with a target speed of  $z_r = r\Delta v$  is simply  $\varphi_\ell \tau_{j|\ell} \nu_{r|\ell,j}/N$ . Hence, the acceleration and deceleration characteristics can be easily incorporated into the formulation of  $p_i$  (17) given in Lemma 3 by substituting the  $z_r$  appearing inside the formulation of  $k_i$  (18) with the discretized version of speed that can be achieved at cell  $c_i$  for a movement epoch that had started at  $c_\ell$  and destined to  $c_j$ . In other words, the  $z_r$  that is appearing inside the formulation of  $k_i$  (18) must be dependent on the locations of starting and destination cells and also acceleration-deceleration characteristics of the vehicle. Thus, using the limiting approach that was applied to derive the result presented in Theorem 1, it can be easily proven that, in order to capture acceleration-deceleration characteristics with the long-run location distribution, it is enough to redefine  $k_X$  (21) by

$$k_X(x) = \int_0^x dx_d \int_x^a dx_s g_X(x_s, x_d, x) + \int_x^a dx_d \int_0^x dx_s g_X(x_s, x_d, x) \quad (67)$$

where

$$g_X(x_s, x_d, x) = f_{X_s}(x_s) f_{X_d|X_s}(x_d|x_s) E[\frac{1}{\tilde{V}} | X_s = x_s, X_d = x_d, X = x] \quad (68)$$

$$E[\frac{1}{\tilde{V}} | X_s = x_s, X_d = x_d, X = x] = \int_{v_{\min}}^{v_{\max}} \frac{dv}{f_\phi(x_s, x_d, v, \phi_{acc}, \phi_{dec}, x)} f_{V|X_s, X_d}(v|x_s, x_d), \quad (69)$$

At this point, we should note that the  $k_X$  defined in (21) is formulated by  $E[\frac{1}{\tilde{V}} | X_s = x_s, X_d = x_d]$  (23), which has the same value for all  $X$  between the points  $X_s$  and  $X_d$ , which doesn't apply to the finite acceleration-deceleration case.

Next, notice that when acceleration-deceleration formulation comes into the picture, since mobile accelerates (decelerates) to (from) target speed  $V$ ,  $\tilde{V}(t)$ , that is, the speed of the mobile at time  $t$ , must be defined on the set  $\{0\} \cup \{v|0 < v \leq v_{\max}\}$ . Therefore, the distribution of  $\tilde{V}$  (i.e., the random variable having the long-run distribution of  $\tilde{V}(t)$ ) can only be determined by considering all possible target speeds  $V \in [v_{\min}, v_{\max}]$  for a given movement epoch between  $X_s$  and  $X_d$ , and checking whether it is possible to have speed  $\tilde{V}$  at a point  $X$  on the path between  $X_s$  and  $X_d$ . As a result, using the formulation of  $\pi_{i,j}$  (12), we obtained the following pdf for  $\tilde{V}$ , which was first defined in Theorem 2 for the infinite acceleration-deceleration case,

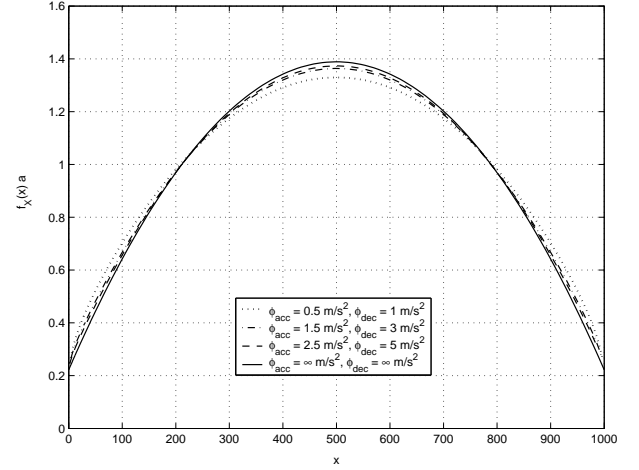


Fig. 7.  $f_X$  for Example 5. ( $a = 1000$  m,  $v_{\min} = 1$  m/s,  $v_{\max} = 20$  m/s,  $E[T_p] = 15$  sec)

$$f_{\tilde{V}}(\tilde{v}) = \begin{cases} \frac{E[T_p|0 \leq X_s \leq a] \delta(\tilde{v})}{E[T_p|0 \leq X_s \leq a] + \bar{D}}, & \text{if } \tilde{v} = 0 \\ \frac{\int_0^a dx k_{\tilde{V}}(x, \tilde{v})}{E[T_p|0 \leq X_s \leq a] + \bar{D}}, & \text{if } \tilde{v} \in (0, v_{\max}] \end{cases}, \quad (70)$$

where  $k_{\tilde{V}}(x, \tilde{v})$  is defined by (33), but the integrand of it, (i.e.,  $g_{\tilde{V}}(x_s, x_d, \tilde{v})$  (33)), is reformulated by

$$g_{\tilde{V}}(x_s, x_d, \tilde{v}, x) = f_{X_s}(x_s) f_{X_d|X_s}(x_d|x_s) \tilde{g}_\phi(x_s, x_d, \tilde{v}, \phi_{acc}, \phi_{dec}, x) \quad (71)$$

where

$$\tilde{g}_\phi(x_s, x_d, \tilde{v}, \phi_{acc}, \phi_{dec}, x) = \int_{v_{\min}}^{v_{\max}} dv f_{V|X_s, X_d}(v|x_s, x_d) \frac{1}{\tilde{v}} \mathbf{1}_{\{\tilde{v} = f_\phi(x_s, x_d, v, \phi_{acc}, \phi_{dec}, x)\}}, \quad (72)$$

which implies

$$E[\tilde{V}] = \frac{\bar{D}}{E[T_p|0 \leq X_s \leq a] + \bar{D}} \quad (73)$$

where  $\bar{D}$  is simply given by (27), and  $\hat{D}$  is again defined by (24), but its integrand is the  $k_X$  formulated above by (67).

At this point, it should be noted that since the function  $f_\phi(X_s, X_d, V, \phi_{acc}, \phi_{dec}, X)$  is determined according to the comparison of the variables  $X_1$  and  $X_2$ , which are defined in terms of  $X_s$ ,  $X_d$ ,  $V$ ,  $\phi_{acc}$ , and  $\phi_{dec}$ , it is very complicated to find closed form expressions for  $k_X$  (67) even for the simplest nontrivial case (i.e., random waypoint mobility model). Therefore, in the following example scenarios, which are presented to demonstrate the effects of different acceleration-deceleration parameters on the long-run location distribution and expected value of speed at the long-run, we evaluated  $k_X$ , and also  $\hat{D}$  using numerical integration methods.

**Example 5:** We now focus on the original random waypoint model (i.e., uniformly selected destination and speed, location independent pause time distribution). Fig. 7 depicts several  $f_X$  that are obtained for different acceleration-deceleration parameters.

First, observe that as  $\phi_{acc}$  and  $\phi_{dec}$  increases, the plot of  $f_X$  gets close to the plot of the case where  $\phi_{acc} = \infty$  and  $\phi_{dec} = \infty$ , which is consistent with the intuitive expectations. Second, for reasonable values of  $\phi_{acc}$  and  $\phi_{dec}$ , the probability of the terminal to be located at the center of the region is lower than the case of infinite acceleration and deceleration.

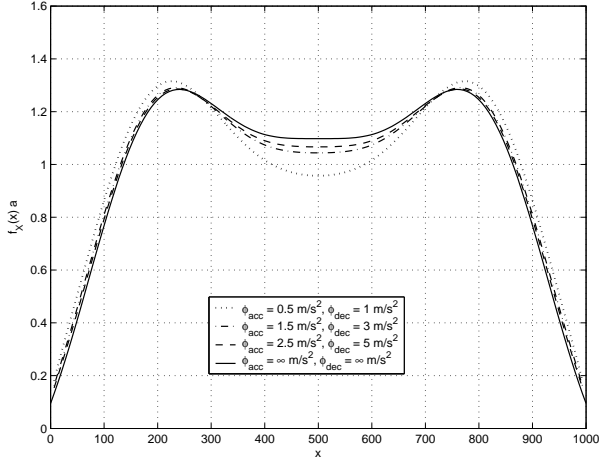


Fig. 8.  $f_X$  for Example 6. ( $a = 1000$  m,  $V$  is uniform in  $[v_{\min}, v_{\max}]$ ,  $v_{\min} = 1$  m/s,  $v_{\max} = 20$  m/s,  $C = 10$  sec)

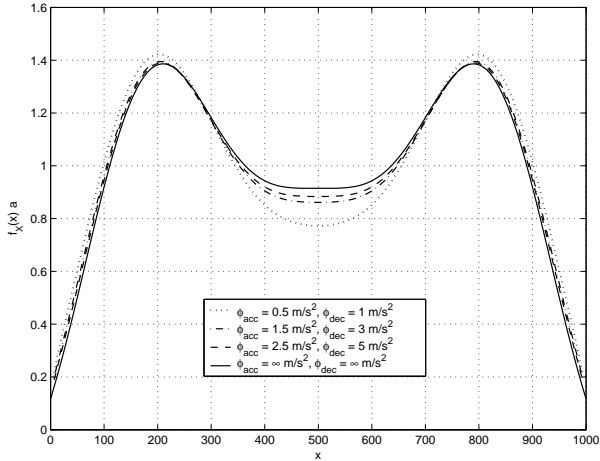


Fig. 9.  $f_X$  for Example 6. ( $a = 1000$  m,  $f_{V|X_s, X_d}$  is given by (40),  $\sigma = 5$ ,  $v_{\min} = 1$  m/s,  $v_{\max} = 20$  m/s,  $C = 10$  sec)

*Example 6:* For this example, we assume that the distribution of  $X_d$  is independent from  $X_s$  and is given by the following sinusoidal function

$$f_{X_d}(x_d) = \frac{3\pi(1 + \sin(3\pi x_d/a))}{a(2 + 3\pi)} \quad (74)$$

which has maximums at the points  $a/6$  and  $5a/6$ , and minimum at  $a/2$ . Furthermore, we assume that

$$E[T_p|X_s = x_s] = aC f_{X_d}(x_s) \quad (75)$$

where  $C > 0$ , which implies  $E[T_p|0 \leq X_s \leq a] \approx 1.31C$ .

Observe that, these mobility characterization parameters can be used model a scenario where mobiles select the destinations around the points  $a/6$  and  $5a/6$  with higher probability, and pause for a longer amount of time around those locations.

In Fig. 8, we assumed  $V$  to be uniformly distributed in  $[v_{\min}, v_{\max}]$ , and plotted  $f_X$  for different acceleration and deceleration parameters. Clearly the first observation that we have made in Example 5 for the effects of the acceleration-deceleration on the random waypoint model is also valid for this scenario. However, it is clear that for this scenario, the difference between finite and infinite acceleration-deceleration cases becomes noticeable as acceleration-deceleration parameters decreases, especially at the center of the region.

For comparison purposes, we also concentrated on the case where  $f_{V|X_s, X_d}$  is defined by the truncated normal distribution

TABLE II  
 $E[\tilde{V}]$  (m/s) FOR EXAMPLE 7 ( $v_{\min}=1$  m/s,  $v_{\max}=20$  m/s)

$\phi_{acc}$ (m/s <sup>2</sup> )	$\phi_{dec}$ (m/s <sup>2</sup> )	$E[T_p]$ (sec)	$E[\tilde{V}]$ $\sigma \rightarrow \infty$	$E[\tilde{V}]$ $\sigma = 10$	$E[\tilde{V}]$ $\sigma = 5$	$E[\tilde{V}]$ $\sigma = 1$
0.5	1	0	5.03	5.31	5.71	6.39
1.5	3	0	5.82	6.15	6.62	7.41
2.5	5	0	6.02	6.37	6.84	7.65
$\infty$	$\infty$	0	6.34	6.70	7.17	8.00
0.5	1	5	4.68	4.92	5.26	5.84
1.5	3	5	5.35	5.63	6.02	6.67
2.5	5	5	5.52	5.81	6.20	6.86
$\infty$	$\infty$	5	5.79	6.09	6.48	7.15
0.5	1	15	4.10	4.26	4.55	4.97
1.5	3	15	4.61	4.82	5.09	5.56
2.5	5	15	4.74	4.95	5.23	5.69
$\infty$	$\infty$	15	4.93	5.15	5.42	5.88

defined in Example 2 by (40). Remember that, since  $\mu(x_s, x_d)$  is linearly dependent to  $|x_d - x_s|$ , the possibility of selecting  $V$  directly proportional to  $|X_d - X_s|$  increases as  $\sigma$  decreases. In Fig. 9, we set  $\sigma = 5$  and plotted  $f_X$  for different acceleration-deceleration parameters. Notice that when  $V$  is proportional to the distance that is going to be traveled (i.e.,  $|X_s - X_d|$ ), the differences between long-run distributions in the neighborhood of the points  $a/6$  and  $5a/6$  becomes less sensitive to the acceleration-deceleration characteristics of vehicles. In addition, long-run proportion of times spent at the locations connecting the points  $a/6$  and  $5a/6$  decreases when  $V$  is proportional to  $|X_s - X_d|$ . These are expected because in this scenario, mobility model do not assign a speed  $V$  for a movement epoch that is impossible to achieve, for example, high speed for a short distance, or a speed that is unrealistically low for long distance. As a result, mobiles spend less proportion of time on the locations connecting hotspots. We also note that, the experiments that are presented by Fig. 9, can be also done for less values of  $\sigma$ . However, since we are evaluating the  $k_X$  and  $\hat{D}$  numerically, the cost of the numerical integration procedures increases as the p.d.f.  $f_{V|X_s, X_d}$  converges to the form given by (45) (i.e., the unit-impulse function at the point  $\mu(x_s, x_d)$ ).

*Example 7:* As a final example, we concentrated on the measure  $E[\tilde{V}]$ , that is, expected speed at the long-run, which is formulated by (73) for the finite acceleration-deceleration parameters. In order to also analyze the case that captures the method of determining  $V$  according to the distance that is going to be traveled, we considered the mobility parameters of Example 2. Recall that, the  $f_{V|X_s, X_d}$  defined by (40) in Example 2 converges to the uniform distribution in  $[v_{\min}, v_{\max}]$  as  $\sigma \rightarrow \infty$ . Hence, we evaluated  $E[\tilde{V}]$  for four different values of  $\sigma$ , and for infinite and various finite acceleration-deceleration parameters. Results are shown in Table II.

As it can be seen in Table II, the values of  $E[\tilde{V}]$  for the finite acceleration-deceleration parameters are always less than their counterparts that are evaluated by assuming acceleration and deceleration to be infinite. Obviously, the difference between them increases as the parameters  $\phi_{acc}$  and  $\phi_{dec}$  decreases. On the other hand, the gap between the  $E[\tilde{V}]$  obtained for the same infinite and finite pairs of  $\phi_{acc}$  and  $\phi_{dec}$  decreases, as  $E[T_p]$  increases, which is expected because the proportion of time  $\tilde{V}$  possesses zero speed also increases. In addition, for given values of  $\phi_{acc}$ ,  $\phi_{dec}$ , and  $E[T_p]$ , a comparison of

the value of  $E[\tilde{V}]$  with its counterpart for the infinite  $\phi_{acc}$ ,  $\phi_{dec}$  case reveals out that the difference between them is more or less the same for all values of  $\sigma$  considered. From this observation, we conclude that if  $D_\phi(X_s, X_d, V)$  denotes the total distance traveled while accelerating and decelerating during a movement epoch between  $X_s$  and  $X_d$  with a target speed of  $V$ , then the proportion  $\frac{D_\phi(X_s, X_d, V)}{|X_s - X_d|}$  averaged over all possible  $X_s$ ,  $X_d$ , and  $V$  is rather insensitive to the choice of  $\sigma$ . Hence, even if  $V$  is determined according to the distance that is going to be traveled with high probability, there will always be periods of acceleration and deceleration that affects the value of  $E[\tilde{V}]$ .

Consequently, the results presented in Table II shows that if a performance measure of interest evaluated for an wireless ad hoc network is dependent on the expected speed at the long run (i.e.,  $E[\tilde{V}]$ ), then the acceleration-deceleration characteristics of the mobile terminals must be captured by the mobility model.

#### IV. CONCLUSIONS

For ad hoc wireless networks, we proposed a generalized random mobility model capable of capturing several scenarios, including hotspots and displacement dependent speed distributions. The analytical framework we presented for the long-run analysis of this generic mobility model over one-dimensional mobility terrains provided closed form expressions for the long-run location and speed distributions. We also provided an extension on our results so that they can be used to examine the effects of acceleration characteristics of vehicles on the long-run location and speed distributions. Our example scenarios verify the usefulness of our analytical framework. Future work will consider the extension of these results to two-dimensional regions.

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